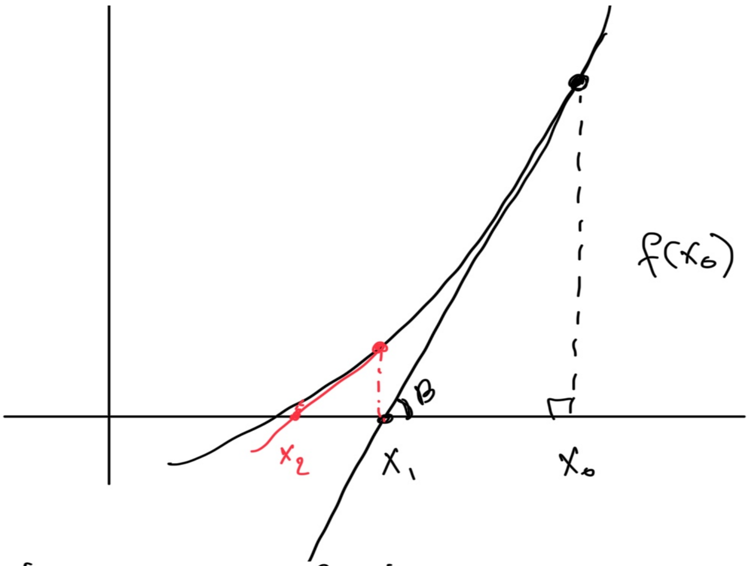
Newton method to solve

$f(x)=0$



$$\tan(B) = f'(x_0) = \frac{f(x_0)}{x_0 - x_1}$$

$$\boxed{x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$\vdots$$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

We can use this Formural to find The root of $f(x)=0$.

Example

Set up a Newton iteration for computing the square root x of a given positive number c and apply it for $c = 2$.

Sol

$$x = \sqrt{c}$$

$$f(x) = x^2 - c = 0$$

$$f'(x) = 2x$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

$$= x_n - \frac{x_n^2 - c}{2x_n}$$

for $c = 2$ let $x_0 = 1$

$$x_1 = 1 - \frac{1-2}{2} = 1.5$$

$$x_2 = 1.5 - \frac{(1.5)^2 - 2}{2} = 1.375$$

$$x_3 = 1.375 - \frac{(1.375)^2 - 2}{2} = 1.43$$

$$x_4 = 1.43 - \frac{(1.43)^2 - 2}{2} = 1.408$$

$\vdots$

Example (3)

Apply NewTon method to

Solve The equation

$f(x) = x + x - 1 = 0$

Sol

$$f(x) = x^3 + x - 1 = 0$$

$$f'(x) = 3x^2 + 1$$

$$x_{n+1} = x_n - \frac{x^3 + x - 1}{3x^2 + 1}$$

$$x_1 = 1 - \frac{1}{4} = 0.75$$

$$x_2 = \quad = 0.686047$$

$$x_3 \quad = 0.6823$$

$$x_4 \quad = 0.68232$$

End of the lecture

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